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DYNAMIC BALANCING OF A NONROLLING, STING SUPPORTED, WIND TUNNEL MODEL PERFORMING NONPLANAR MOTIONS

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HYPERSONIC RESEARCH LABORATORY

PROJECT 7064

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AIR FORCE SYSTEMS COMMAND
United States Air Force

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UNITED STATES AIR FORCE
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

FOREWORD

This report was prepared by Capt Kevin Yelmgren of the Aerospace Research Laboratories, Air Force Systems Command, under Project 7064, entitled "High Velocity Fluid Mechanics."

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ABSTRACT

Wind-off, bench tests on a wind tunnel model that had freedom to pitch, and yaw, but no freedom to roll indicated that the pitching and yawing motions were inertially coupled. An order of magnitude analysis showed that this coupling was due to the product of inertia term I_{yz}. The inertially coupled equations of motion are solved, and a method is presented for determining the magnitude and sign of I_{yz} from recordings of the model motion. The method used to dynamically balance the model is described, and experimental results are presented that verify this method.

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NOMENCLATURE

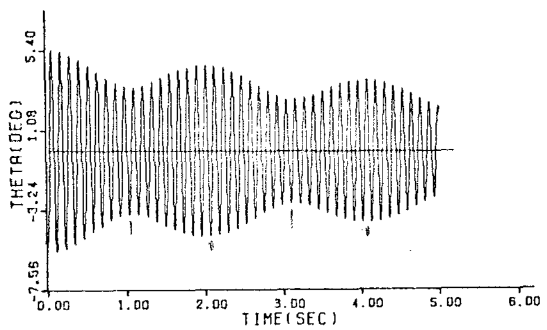
$A_\theta, B_\theta, A_\psi, B_\psi$	Amplitude components
C_θ, C_ψ	Flexure damping constants
I^2	Constant defined by Eq. (19)
I_x, I_y, I_z	Moments of inertia about body axes
I_{xy}, I_{xz}, I_{yz}	Products of inertia about body axes
M_x, M_y, M_z	Moments about body axes
p, q, r	Angular velocity components on body axes system
q_1, q_2	Normal coordinates, Eq. (A-27) and (A-28)
t	Time
x, y, z	Body axes system (Fig. 1)
x_T, y_T, z_T	Space fixed axes
$\delta_\psi, \delta_\theta$	Constants defined by Eqs. (A-38) and (A-39)
ψ, θ, ϕ	Eulerian angles
$\lambda_\theta, \lambda_\psi$	Constants defined by Eqs. (14) and (15)
$\phi_{11}, \phi_{12}, \widetilde{\phi}$	Constants defined by Eqs. (16), (17) and (18)
$\omega_\theta, \omega_\psi$	Circular frequency in pitch and yaw

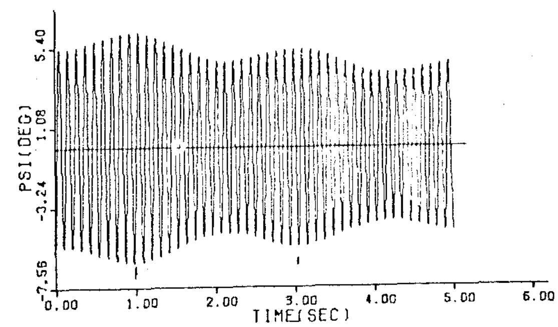
I. INTRODUCTION

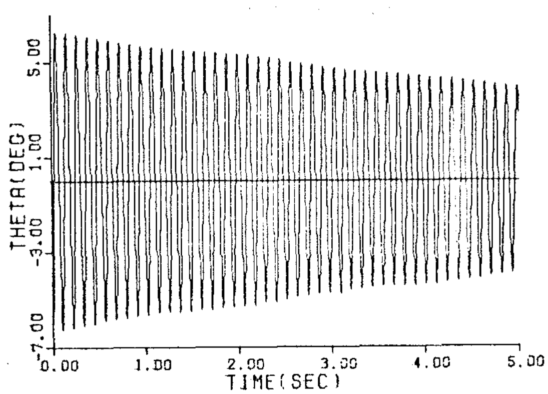
In order to investigate possible aerodynamic interactions between the pitching and yawing motions of a nonrolling body undergoing nonplanar motions, a flexure was designed to give a wind tunnel model freedom to either pitch, yaw or pitch and yaw simultaneously. The model, a 10° half angle cone, was weighted so that the body axes of rotation were intended to be the principal axes of inertia, and yet I_y was not equal to I_z. Releasing a model balanced in this manner from a combined pitch and yaw attitude with zero initial velocities results in a nonplanar Lissajous motion, that is, a motion with different frequencies in pitch and yaw. As the model is not rolling, the pitch and yaw components of the motion should be uncoupled, damped, sinusoidal oscillations.

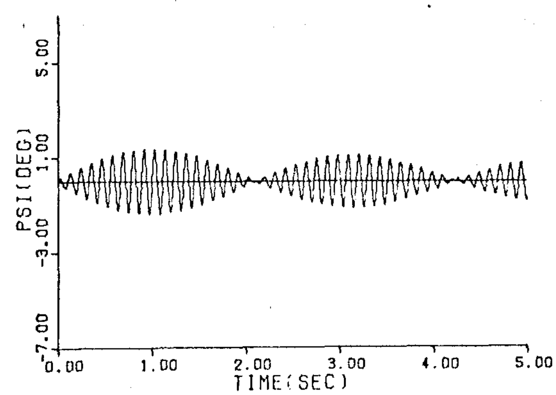
Wind-off bench tests with the model in vacuum did not give the uncoupled motion that was expected. The pitching and yawing motion envelopes were not smoothly damped exponentials but rather "sinusoidally" damped exponentials. The sketches below are an exaggeration of the motion obtained. The initial velocities in both sets of sketches are zero ($\dot{\Theta}_o = \dot{\psi}_o = 0$), with $\psi_o = \Theta_o$ in the first set, and $\psi_o = 0$, in the second set. This beating type of motion indicates that the pitch and yaw motions are coupled due to a slight inertia asymmetry, that is, the principal axes of inertia do not coincide with the axes of rotation. The purpose of this report is to determine which product of inertia causes the coupling, to determine its magnitude from the

observed coupled motions, and finally to show how the model must be balanced to eliminate the inertial coupling.









II. THEORETICAL ANALYSIS

The moment equations of motion for a body fixed coordinate system (Fig. 1), with its origin located at the c. g. of the body, can be written as[1]

$$M_x = I_x\dot{p} + (I_z - I_y)rq - I_{xz}(\dot{r} + qp) + I_{xy}(rp - \dot{q}) + I_{yz}(r^2 - q^2) \tag{1}$$

$$M_y = I_y\dot{q} + (I_x - I_z)pr - I_{xy}(\dot{p} + qr) + I_{yz}(pq - \dot{r}) + I_{xz}(p^2 - r^2) \tag{2}$$

$$M_z = I_z\dot{r} + (I_y - I_x)qp - I_{yz}(\dot{q} + pr) + I_{xz}(qr - \dot{p}) + I_{xy}(q^2 - p^2) \tag{3}$$

where p, q, r are the components of the angular velocity along the body axes, x, y, z respectively. The angular velocity components can be written as

$$p = \dot{\phi} - \dot{\psi}\sin\theta \tag{4a}$$

$$q = \dot{\theta}\cos\phi + \dot{\psi}\cos\theta\sin\phi \tag{4b}$$

$$r = \dot{\psi}\cos\theta\cos\phi - \dot{\theta}\sin\phi \tag{4c}$$

The terms M_x, M_y, M_z are the moments acting on the body. For a nonrolling vehicle in a vacuum, without wind, these terms represent the flexure moments acting on the system. The pitch and yaw moments can be written as

$$M_y = -C_\theta\dot{\theta} - k_\theta\theta \tag{5a}$$

$$M_z = -C_\psi\dot{\psi} - k_\psi\psi \tag{5b}$$

For $\phi = \dot{\phi} = 0$ and assuming small angles, Eq. (4) reduces to

$$p \approx - \dot{\psi}\,\theta \tag{6a}$$

$$q = \dot{\theta} \tag{6b}$$

$$r \approx \dot{\psi} \tag{6c}$$

Substituting Eq. (6) into Eqs. (2) and (3) gives:

$$\begin{aligned} M_y = {} & I_y\ddot{\theta} + (I_x - I_z)(-\dot{\psi}\dot{\psi}\theta) - I_{xy}(-\ddot{\psi}\theta) + I_{yz}(-\dot{\psi}\dot{\theta}\,\theta - \ddot{\psi}) \\ & + I_{xz}(-\dot{\psi}^2\theta^2 - \dot{\psi}^2) \end{aligned} \tag{7a}$$

$$\begin{aligned} M_z = {} & I_z\ddot{\psi} + (I_y - I_x)(-\dot{\psi}\dot{\theta}\,\theta) - I_{yz}(\ddot{\theta} - \dot{\psi}\dot{\psi}\theta) \\ & + I_{xz}(\dot{\theta}\,\dot{\psi} + \ddot{\psi}\theta + \dot{\psi}\,\dot{\theta}) + I_{xy}(\dot{\theta}^2 - \dot{\psi}^2\theta^2) \end{aligned} \tag{7b}$$

These equations can be simplified due to the small angle assumption. Assume that pitch and yaw angles can be approximated as

$$\theta = \theta_o \cos \omega_\theta t, \quad \psi = \psi_o \cos \omega_\psi t$$

Then,

$$\dot{\theta} = - \omega_\theta \theta_o \sin \omega_\theta t\ , \quad \dot{\psi} = - \omega_\psi \psi_o \sin \omega_\psi t$$

and

$$\ddot{\theta} = - \omega_\theta^2 \theta_o \cos \omega_\theta t\ , \quad \ddot{\psi} = - \omega_\psi^2 \psi_o \cos \omega_\psi t$$

An order of magnitude analysis can now be made for the individual terms of Eq. (7)

$$\ddot{\psi}\,\theta \approx \psi_o\,\theta_o\,\omega_\psi^2$$

$$\dot{\psi}\,\dot{\theta} \approx \psi_o\,\theta_o\,\omega_\psi\omega_\theta$$

$$\dot{\theta}\,\dot{\psi}\,\theta \approx \psi_o\,\theta_o^2\omega_\psi\omega_\theta$$

$$\dot{\psi}^2\theta \approx \psi_o^2\,\theta_o\,\omega_\psi^2$$

$$\dot{\psi}^2 \approx \psi_o^2\omega_\psi^2$$

$$\dot{\theta}^2 \approx \theta_o^2\omega_\theta^2$$

$$\ddot{\theta} \approx \theta_o\omega_\theta^2$$

$$\ddot{\psi} \approx \psi_o\omega_\psi^2$$

$$\dot{\psi}^2\theta^2 \approx \psi_o^2\,\theta_o^2\omega_\psi^2$$

All the product terms are small compared to either the $\ddot{\theta}$ or $\ddot{\psi}$ term if the amplitudes ψ_o and θ_o are small. Thus for small angles, Eqs. (7a) and (7b) reduce to

$$M_y = I_y\ddot{\theta} - I_{yz}\ddot{\psi} \tag{8a}$$

$$M_z = I_z\ddot{\psi} - I_{yz}\ddot{\theta} \tag{8b}$$

As can be seen from Eqs. (8a) and (8b), the product of inertia term I_{yz} causes the coupling between the pitch and yaw motions. Although I_{xz} and I_{xy} may be of the same order of magnitude as I_{yz}, they are multiplied by relatively small quantities so that they can be ignored compared to the terms $I_{yz}\ddot{\theta}$ and $I_{yz}\ddot{\psi}$.

The effect of I_{yz} on the pitch and yaw motion and the value of I_{yz} causing the coupling can be determined from the solution of Eqs. (8a) and

(8b). Substituting Eq. (5) into Eq. (8) gives

$$I_y \ddot{\theta} - I_{yz}\ddot{\psi} = - C_\theta \dot{\theta} - k_\theta \theta \tag{9a}$$

$$I_z \ddot{\psi} - I_{yz}\ddot{\theta} = - C_\psi \dot{\psi} - k_\psi \psi \tag{9b}$$

or, in matrix form,

$$\begin{bmatrix} I_y & -I_{yz} \\ -I_{yz} & I_z \end{bmatrix} \begin{Bmatrix} \ddot{\theta} \\ \ddot{\psi} \end{Bmatrix} + \begin{bmatrix} C_\theta & 0 \\ 0 & C_\psi \end{bmatrix} \begin{Bmatrix} \dot{\theta} \\ \dot{\psi} \end{Bmatrix} + \begin{bmatrix} k_\theta & 0 \\ 0 & k_\psi \end{bmatrix} \begin{Bmatrix} \theta \\ \psi \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{10}$$

Equation (10) is a second order, coupled, differential equation. As the coefficient matrices are symmetric, this equation can be solved using a normal coordinate system[2, 3]. The initial conditions used to solve Eq. (10) are

$$\begin{aligned} \theta(0) &= \theta_o \qquad & \psi(0) &= \psi_o \\ \dot{\theta}(0) &= \dot{\theta}_o \qquad & \dot{\psi}(0) &= \dot{\psi}_o \end{aligned} \tag{11}$$

Following a method identical to that given in Ref. 3, the solution to Eq. (10) is derived in Appendix A. The results are

$$\begin{aligned} \theta(t) = {} & e^{-\lambda_\theta t} \left\{ \frac{\phi_{12}}{\tilde{\phi}} (\phi_{11}\psi_o - \theta_o) \cos \omega_\theta t + \frac{\phi_{12}}{\tilde{\phi}} \left[\frac{(\phi_{11}\dot{\psi}_o - \dot{\theta}_o)}{\omega_\theta} + \frac{\lambda_\theta}{\omega_\theta} (\phi_{11}\psi_o - \theta_o) \right] \sin \omega_\theta t \right\} \\ & + e^{-\lambda_\psi t} \left\{ \frac{\phi_{11}}{\tilde{\phi}} (\theta_o - \phi_{12}\psi_o) \cos \omega_\psi t + \frac{\phi_{11}}{\tilde{\phi}} \left[\frac{(\dot{\theta}_o - \phi_{12}\dot{\psi}_o)}{\omega_\psi} + \frac{\lambda_\psi}{\omega_\psi} (\theta_o - \phi_{12}\psi_o) \right] \sin \omega_\psi t \right\} \end{aligned} \tag{12}$$

and,

$$\begin{aligned} \psi(t) = {} & e^{-\lambda_\psi t} \left\{ \frac{(\theta_o - \phi_{12}\psi_o)}{\tilde{\phi}} \cos \omega_\psi t + \frac{1}{\tilde{\phi}} \left[\frac{(\dot{\theta}_o - \phi_{12}\dot{\psi}_o)}{\omega_\psi} + \frac{\lambda_\psi}{\omega_\psi} (\theta_o - \phi_{12}\psi_o) \right] \sin \omega_\psi t \right\} \\ & + e^{-\lambda_\theta t} \left\{ \frac{(\phi_{11}\psi_o - \theta_o)}{\tilde{\phi}} \cos \omega_\theta t + \frac{1}{\tilde{\phi}} \left[\frac{(\phi_{11}\dot{\psi}_o - \dot{\theta}_o)}{\omega_\theta} + \frac{\lambda_\theta}{\omega_\theta} (\phi_{11}\psi_o - \theta_o) \right] \sin \omega_\theta t \right\} \end{aligned} \tag{13}$$

where

$$\lambda_\theta = \frac{\phi_{12}^2 C_\theta + C_\psi}{\phi_{12}^2 k_\theta + k_\psi} \frac{\omega_\theta^2}{2} \approx \frac{C_\theta \omega_\theta^2}{2k_\theta} \tag{14}$$

$$\lambda_\psi = \frac{\phi_{11}^2 C_\theta + C_\psi}{\phi_{11}^2 k_\theta + k_\psi} \frac{\omega_\psi^2}{2} \approx \frac{C_\psi \omega_\psi^2}{2k_\psi} \tag{15}$$

$$\phi_{11} = \frac{I_{yz}\omega_\psi^2}{I_y \omega_\psi^2 - k_\theta} = \frac{I_z \omega_\psi^2 - k_\psi}{I_{yz}\omega_\psi^2} \tag{16}$$

$$\phi_{12} = \frac{I_{yz}\omega_\theta^2}{I_y \omega_\theta^2 - k_\theta} = \frac{I_z \omega_\theta^2 - k_\psi}{I_{yz}\omega_\theta^2} \tag{17}$$

$$\tilde{\phi} = \phi_{11} - \phi_{12} \tag{18}$$

$$I^2 = I_y I_z - (I_{yz})^2 \tag{19}$$

$$\omega_{\psi,\theta}^2 = \left\{\frac{k_\theta I_z + k_\psi I_y}{2I^2}\right\} \pm \sqrt{\left\{\frac{k_\theta I_z + k_\psi I_y}{2I^2}\right\}^2 - \frac{k_\theta k_\psi}{I^2}} \tag{20}$$

The sign in front of the square root term in Eq. (20) is determined from the case of zero I_{yz}, where $\omega_\psi^2 = k_\psi / I_z$, and $\omega_\theta^2 = k_\theta / I_y$. If k_ψ / I_z is greater than k_θ / I_y then the plus sign in Eq. (20) will be associated with ω_ψ^2, and the minus sign with ω_θ^2. If k_θ / I_y is greater than k_ψ / I_z, then the reverse will be true.

The effect of I_{yz} on the model motion can be seen from Eqs. (12), and (13). As there are two, nearly equal, frequency components present in each equation, the resulting motion for both pitch and yaw will be a beating type of motion. That is, the envelopes will have a sinusoidal type pattern.

The effect of I_{yz} on the frequencies can be determined from Eq. (20), which can be rewritten as

$$\left(\frac{\omega_\theta}{k_\theta/I_y}\right)^2 = \frac{(I_z/I_y + k_\psi/k_\theta)}{2(I_z/I_y - I_{yz}^2/I_y^2)} - \sqrt{\left(\frac{I_z/I_y + k_\psi/k_\theta}{2(I_z/I_y - I_{yz}^2/I_y^2)}\right)^2 - \frac{k_\psi/k_\theta}{(I_z/I_y - I_{yz}^2/I_y^2)}} \tag{21}$$

and,

$$\left(\frac{\omega_\psi}{k_\psi/I_z}\right)^2 = \frac{I_z/I_y + \dfrac{k_\theta I_z^2}{k_\psi I_y^2}}{2(I_z/I_y - I_{yz}^2/I_y^2)} + \sqrt{\left\{\frac{I_z/I_y + \dfrac{k_\theta I_z^2}{k_\psi I_y^2}}{2(I_z/I_y - I_{yz}^2/I_z^2)}\right\}^2 - \frac{\dfrac{k_\theta I_z^2}{k_\psi I_y^2}}{(I_z/I_y - I_{yz}^2/I_y^2)}} \tag{22}$$

Equations (21) and (22) are plotted in Figs. 2 and 3, as the frequency ratios $\omega_\theta/(k_\theta/I_y)^{1/2}$ and $\omega_\psi/(k_\psi/I_z)^{1/2}$ versus I_{yz}/I_y for k_ψ/k_θ equal to 1.014, and for several values of I_z/I_y. These curves show that I_{yz} has a small effect on the frequency, causing it to increase or decrease depending on the value of I_z/I_y.

The moments of inertia, I_y and I_z, as well as the product of inertia, I_{yz}, can be obtained from Eqs. (16) to (18) as

$$I_y = \frac{k_\theta}{\tilde{\phi}} \frac{\phi_{11}\omega_\theta^2 - \phi_{12}\omega_\psi^2}{\omega_\psi^2\omega_\theta^2} \tag{23}$$

$$I_z = \frac{k_\psi}{\tilde{\phi}} \frac{\phi_{11}\omega_\psi^2 - \phi_{12}\omega_\theta^2}{\omega_\psi^2\omega_\theta^2} \tag{24}$$

$$I_{yz} = \frac{k_\psi}{\tilde{\phi}} \frac{(\omega_\psi^2 - \omega_\theta^2)}{\omega_\psi^2\omega_\theta^2} \tag{25}$$

In order to determine the I_{yz} of a system, values of k_ψ, $\tilde{\phi}$, ω_ψ and ω_θ are required. While the flexure stiffness, k_ψ, is known from static bench tests, ω_θ, ω_ψ, and $\tilde{\phi}$ must be determined from dynamic tests, i.e., from recordings of the pitching and yawing motions.

Values for ϕ_{11} and ϕ_{12} can be determined as follows. Assuming that the initial angular velocities are zero, Eqs. (12) and (13) reduce to

$$\theta(t) = e^{-\lambda_\theta t}\left\{\frac{\phi_{12}}{\tilde{\phi}}(\phi_{11}\psi_o - \theta_o)\cos\omega_\theta t + \frac{\phi_{12}\lambda_\theta}{\tilde{\phi}\omega_\theta}(\phi_{11}\psi_o - \theta_o)\sin\omega_\theta t\right\}$$

$$+ e^{-\lambda_\psi t}\left\{\frac{\phi_{11}}{\tilde{\phi}}(\theta_o - \phi_{12}\psi_o)\cos\omega_\psi t + \frac{\phi_{11}\lambda_\psi}{\tilde{\phi}\omega_\psi}(\theta_o - \phi_{12}\psi_o)\sin\omega_\psi t\right\}$$

and,

$$\psi(t) = e^{-\lambda_\psi t}\left\{\frac{(\theta_o - \phi_{12}\psi_o)}{\tilde{\phi}}\cos\omega_\psi t + \frac{\lambda_\psi}{\tilde{\phi}\omega_\psi}(\theta_o - \phi_{12}\psi_o)\sin\omega_\psi t\right\}$$

$$+ e^{-\lambda_\theta t}\left\{\frac{(\phi_{11}\psi_o - \theta_o)}{\tilde{\phi}}\cos\omega_\theta t + \frac{\lambda_\theta}{\tilde{\phi}\omega_\theta}(\phi_{11}\psi_o - \theta_o)\sin\omega_\theta t\right\}$$

or,

$$\theta(t) = A_\theta e^{-\lambda_\theta t}\cos(\omega_\theta t - \gamma_{\theta\theta}) + B_\theta e^{-\lambda_\psi t}\cos(\omega_\psi t - \gamma_{\theta\psi}) \tag{26}$$

$$\psi(t) = A_\psi e^{-\lambda_\psi t}\cos(\omega_\psi t - \gamma_{\psi\psi}) + B_\psi e^{-\lambda_\theta t}\cos(\omega_\theta t - \gamma_{\psi\theta}) \tag{27}$$

where

$$A_\theta = \phi_{12}\frac{(\phi_{11}\psi_o - \theta_o)}{\tilde{\phi}}\sqrt{1 + \frac{\lambda_\theta^2}{\omega_\theta^2}} \tag{28a}$$

$$B_\theta = \frac{\phi_{11}}{\tilde{\phi}}(\theta_o - \phi_{12}\psi_o)\sqrt{1 + \frac{\lambda_\psi^2}{\omega_\psi^2}} \tag{28b}$$

$$A_\psi = \frac{(\theta_o - \phi_{12}\psi_o)}{\tilde{\phi}} \sqrt{1 + \frac{\lambda_\psi^2}{\omega_\psi^2}} \tag{29a}$$

$$B_\psi = \left(\frac{\phi_{11}\psi_o - \theta_o}{\tilde{\phi}}\right) \sqrt{1 + \frac{\lambda_\theta^2}{\omega_\theta^2}} \tag{29b}$$

$$\tan\gamma_{\theta\theta} = \tan\gamma_{\psi\theta} = \frac{\lambda_\theta}{\omega_\theta}$$

$$\tan\gamma_{\psi\psi} = \tan\gamma_{\theta\psi} = \frac{\lambda_\psi}{\omega_\psi}$$

therefore, from Eqs. (28) and (29)

$$\phi_{11} = \frac{B_\theta}{A_\psi} \, , \qquad \phi_{12} = \frac{A_\theta}{B_\psi} \tag{30}$$

The sign of I_{yz} is associated with the sign of $\tilde{\phi}$ which depends upon the signs of the A's and B's. If the envelope of the motion initially decreases, then A and B in Eq. (26) or (27) will have the same sign. However, if the envelope initially increases, then A and B will have opposite signs. The sign of A always has the same sign as the initial release angle.

III. EXPERIMENTAL INVESTIGATION

The objective of the experimental investigation was to determine the I_{yz} of the model, and then to eliminate it by properly balancing the model. The I_{yz} of the original model was small, and therefore it was difficult to determine the amplitude components of the motion. In order to overcome this difficulty, a larger, known I_{yz} was added to the model. The resulting motion was more easily reducible. The original I_{yz} of the model, plus the added I_{yz} was determined by fitting the data to Eqs. (26) and (27) by the least squares differential corrections technique[4,5]. The added I_{yz} was then subtracted to give the I_{yz} of the model. This was repeated with another known I_{yz}, and the model I_{yz} was taken as the average of the two results.

PROCEDURE

The two model configurations tested were the basic model plus:

Configuration	$I_{yz\ added}$ sl-ft^2	$I_{y\ added}$ sl-ft^2	$I_{z\ added}$ sl-ft^2
(1)	-5.25×10^{-5}	4.843×10^{-4}	5.393×10^{-4}
(2)	$+4.27 \times 10^{-5}$	4.843×10^{-4}	5.393×10^{-4}

For each model configuration the model was first balanced in the xz, and yz planes to insure that the center of gravity was located at the pivot point. The model was mounted in a test cabin which was evacuated to less than 2mm of mercury to eliminate aerodynamic effects. The model was set at a predetermined pitch and yaw angle, and released.

The resulting angular motion was recorded by strain gauges mounted on the flexure. The analogue output from the gauges was digitized, and stored on magnetic tapes. Calibration runs for the conversion of the digitized data to degrees were made before and after each run. The pitch data was fitted to Eq. (26), and the yaw data to Equation (27). The section length fitted was one beat period. The amplitude components determined from the least squares fit were then used to calculate ϕ_{11}, and ϕ_{12}, Eq (30), and then $\tilde{\phi}$, Equation (18). The value of $\tilde{\phi}$, along with ω_θ, and ω_ψ, which were also obtained from the least squares fit, were used to calculate I_{yz}, Eq. (25), as well as I_y, Eq. (23), and I_z, Equation (24). Three consecutive sections were fitted, and the average values resulting from these three fits were taken as the I_{yz}, I_y, and I_z of the configuration. The added values of I_{yz}, I_y and I_z were subtracted from the measured values to give I_{yz}, I_y and I_z of the original model.

RESULTS

The motions that were obtained from the runs are shown in Figs. 4, 5, and 6. As can be seen from run #1, Figs. 4a and 4b, the θ envelope initially increases while the ψ envelope initially decreases. The I_{yz} of this configuration should be negative. This is in accord with the $I_{yz} = -5.25 \times 10^{-5}$ sl-ft^2, that was added to the model. For run #2, Figs. 5a, and 5b, the situation is reversed. The θ envelope initially decreases, while the ψ envelope initially increases. The I_{yz} of this configuration

should be positive. This is again in agreement with the $I_{yz} = +4.27 \times 10^{-5}$ $sl\text{-}ft^2$ that was added to the model for this run.

The results of the least squares fitting procedure are summarized in Table I.

TABLE I: Results From Least Squares Fit to Data

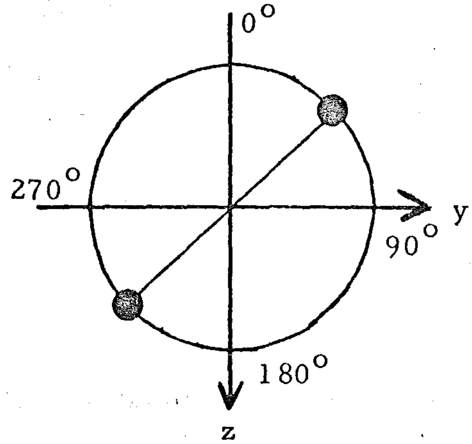
Configuration	$I_{y\,model}$ $sl\text{-}ft^2$	$I_{z\,model}$ $sl\text{-}ft^2$	$I_{yz\,model}$ $sl\text{-}ft^2$
1	0.002732	0.002397	0.30×10^{-5}
2	0.002737	0.002405	0.37×10^{-5}

The average value for the I_{yz} of the model is 0.34×10^{-5} $sl\text{-}ft^2$ with a probable error of $\pm 0.11 \times 10^{-5}$ $sl\text{-}ft^2$, or about 32% of the average value. The average values for I_y and I_z are 0.002735 $sl\text{-}ft^2$, and 0.002401 $sl\text{-}ft^2$, respectively. The probable error of these terms is estimated to be about 0.1%.

The angle between the principal axes, and the body axes is given by $\frac{1}{2}\tan^{-1}\left(\frac{2I_{yz}}{I_z - I_y}\right)$, which for small angles reduced to $\frac{I_{yz}}{I_z - I_y}$. For runs 1 and 2, with intentionally added I_{yz}'s (Figs. 4 and 5), this angle is 10.2 degrees and 9.6 degrees respectively. For the original model, the angle is 0.6 degrees. Even this small angle of less than one degree was sufficient to cause inertial coupling of the pitching and yawing motion that could be detected on oscillograph records.

IV. DYNAMIC BALANCING METHOD

The I_{yz} of the model was determined to be 0.34×10^{-5} sl-ft². An I_{yz} of -0.32×10^{-5} sl-ft² was then added to the model by placing equal masses at 45° and 225° on the model (see sketch). This should reduce the net I_{yz}



of the model to approximately zero. The model was statically balanced by placing masses on the geometric, or principal axes, i.e., 0°, 90°, 180°, or 270°. This prevented any additional I_{yz} being added to the model. This configuration was run, and the resulting motion, Figs. 6a and 6b, showed that the model is in near perfect trim.

V. CONCLUSION

The present study of the motion in vacuum of a nonrolling body with freedom to pitch and yaw shows that the product of inertia I_{yz} is the dominant term causing inertial coupling of the pitching and yawing motion. The presented analysis, and the described method of eliminating the inertial coupling have been verified by experiments. The model is now ready for wind tunnel tests, the results of which will be reported later.

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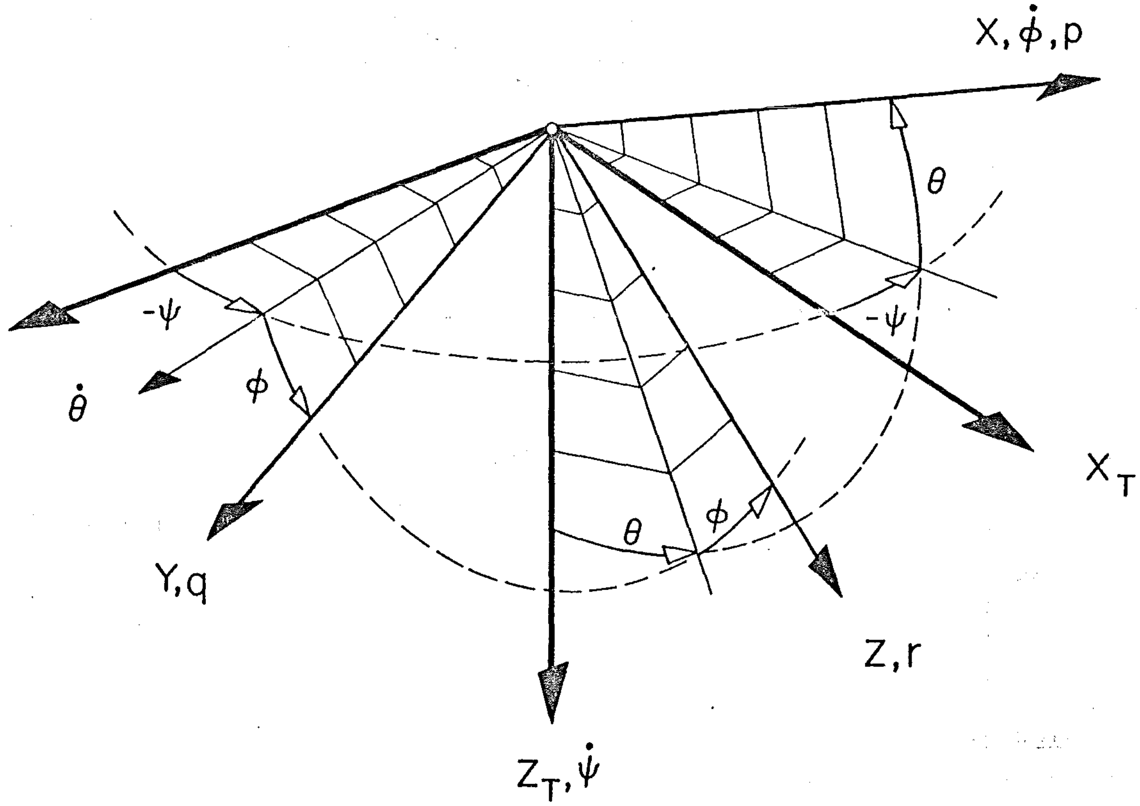


FIGURE 1 Coordinate system

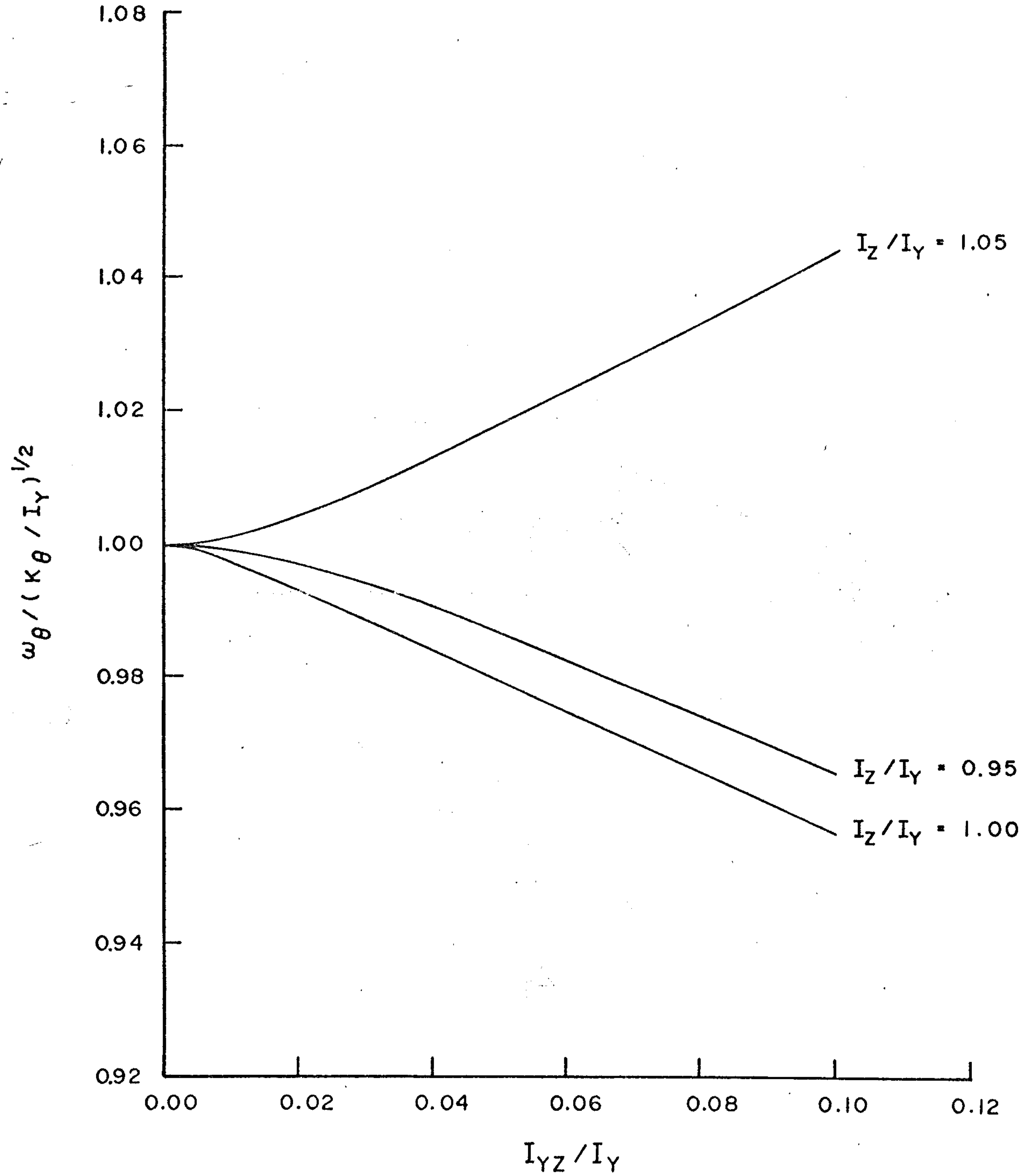


FIGURE 2 Effect of I_{yz} on circular frequency in pitch

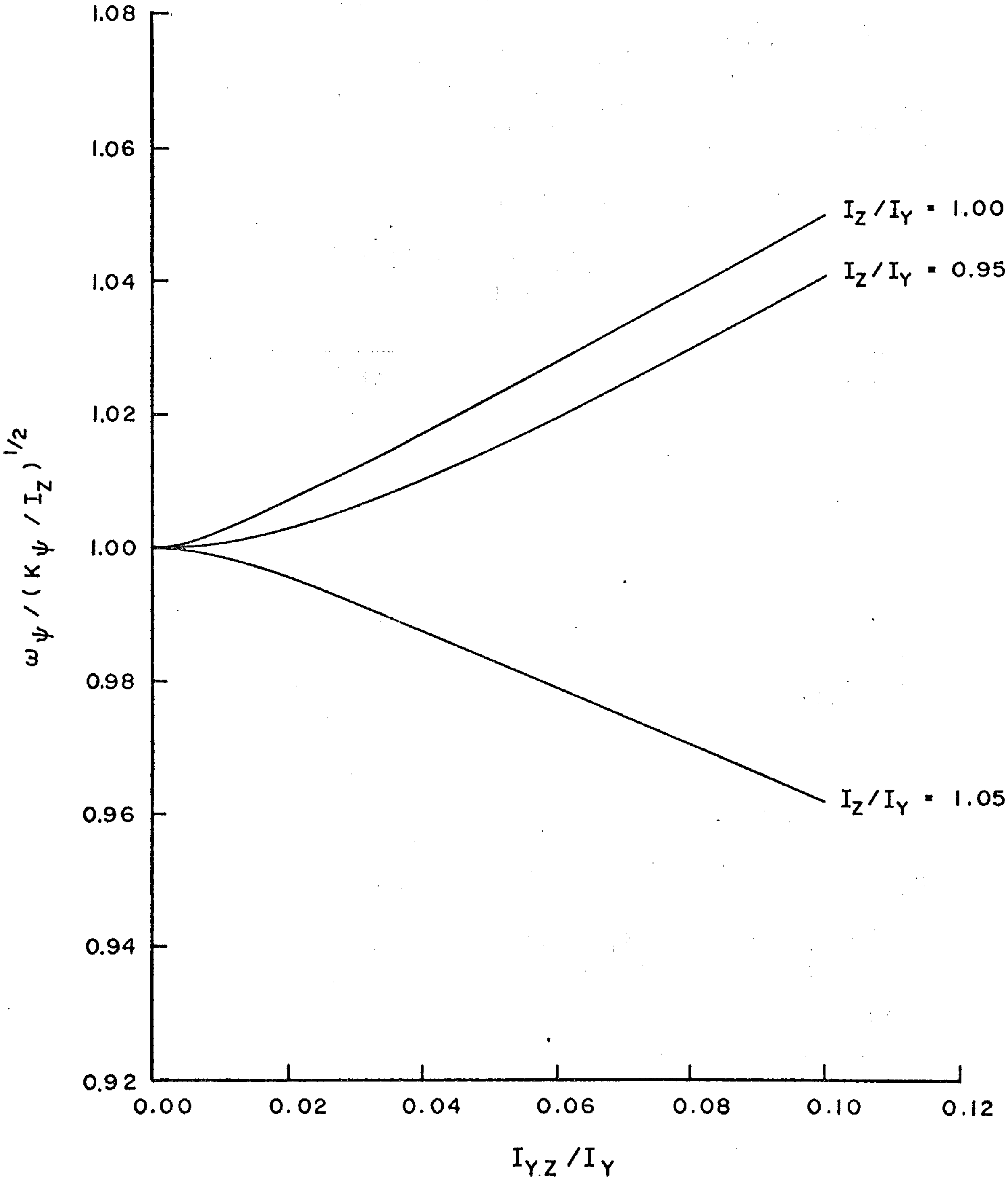
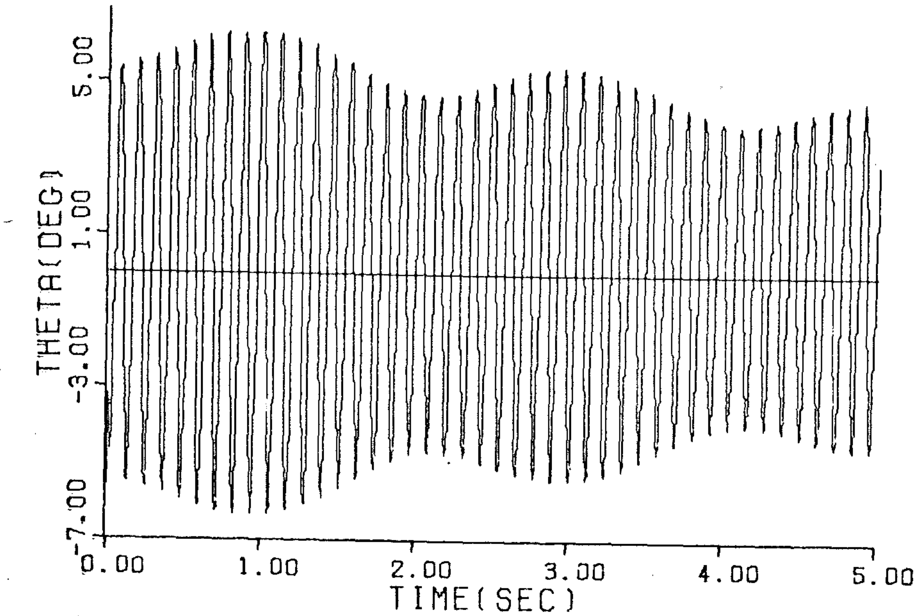
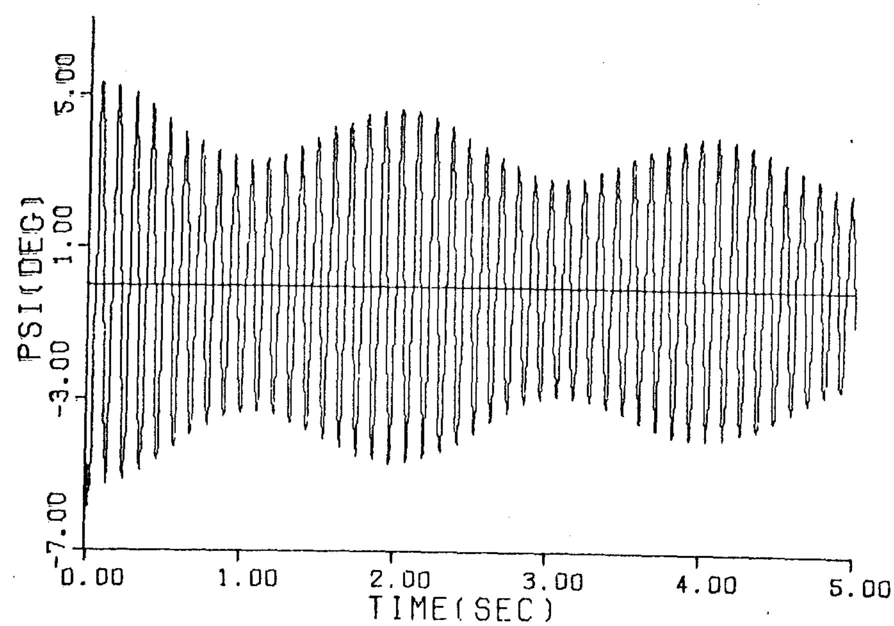


FIGURE 3 Effect of I_{yz} on circular frequency in yaw

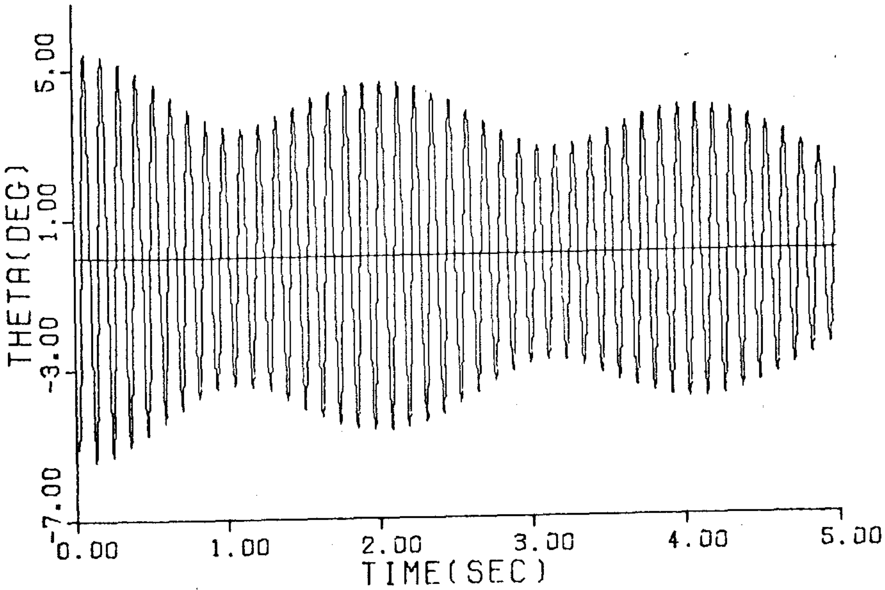


a. Pitch component vs time

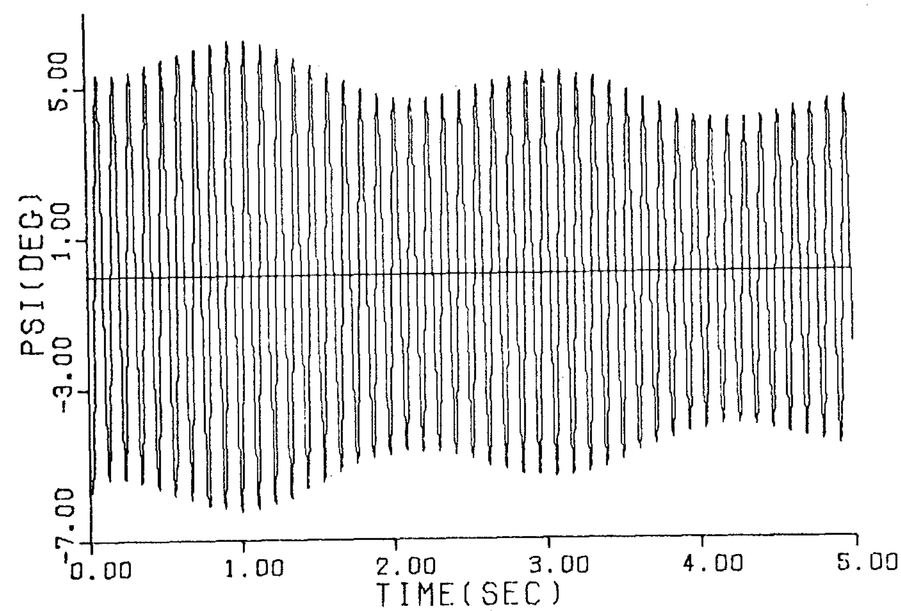


b. Yaw component vs time

FIGURE 4 Pitch and yaw components for original model plus an added I_{yz} of -5.25×10^{-5} slug-ft^2

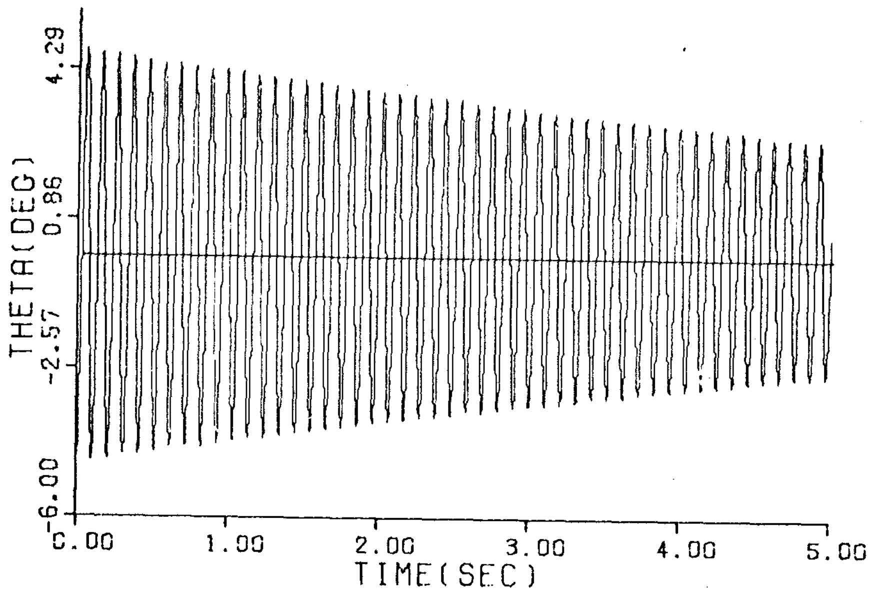


a. Pitch component vs time

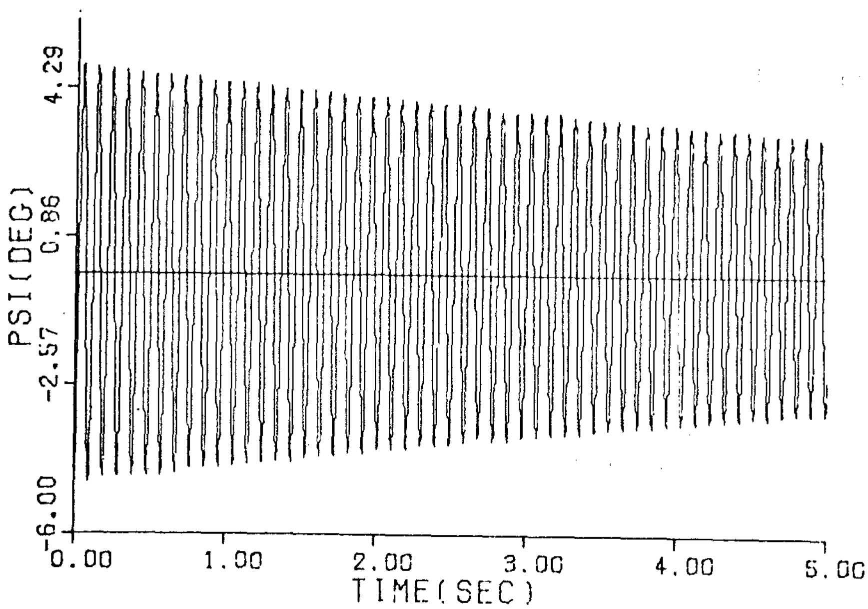


b. Yaw component vs time

FIGURE 5 Pitch and yaw components for original model plus an added I_{yz} of $+ 4.27 \times 10^{-5}$ slug-ft^2



a. Pitch component vs time.



b. Yaw component vs time

FIGURE 6 Pitch and yaw components for dynamically balanced model

VII. APPENDIX A

The governing equations of motion in a body fixed coordinate system with its origin located at the c. g. are

$$I_y\ddot{\theta} - I_{yz}\ddot{\psi} + C_\theta\dot{\theta} + k_\theta\theta = 0 \tag{A-1}$$

$$-I_{yz}\ddot{\theta} + I_z\ddot{\psi} + C_\psi\dot{\psi} + k_\psi\psi = 0 \tag{A-2}$$

Consider first the undamped equations of motion

$$I_y\ddot{\theta} - I_{yz}\ddot{\psi} + k_\theta\theta = 0 \tag{A-3}$$

$$-I_{yz}\ddot{\theta} + I_z\ddot{\psi} + k_\psi\psi = 0 \tag{A-4}$$

In matrix form, Eqs. (A-3) and (A-4) are written as

$$\begin{bmatrix} I_y & -I_{yz} \\ -I_{yz} & I_z \end{bmatrix}\begin{Bmatrix} \ddot{\theta} \\ \ddot{\psi} \end{Bmatrix} + \begin{bmatrix} k_\theta & 0 \\ 0 & k_\psi \end{bmatrix}\begin{Bmatrix} \theta \\ \psi \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{A-5}$$

One possible solution has the form

$$\begin{Bmatrix} \theta \\ \psi \end{Bmatrix} = \begin{Bmatrix} A_1 \\ A_2 \end{Bmatrix} \sin \omega t \tag{A-6}$$

Substituting Eq. (A-6) into Eq. (A-5) gives

$$\begin{bmatrix} I_y & -I_{yz} \\ -I_{yz} & I_z \end{bmatrix}\begin{Bmatrix} -A_1\omega^2 \sin\omega t \\ -A_2\omega^2 \sin\omega t \end{Bmatrix} + \begin{bmatrix} k_\theta & 0 \\ 0 & k_\psi \end{bmatrix}\begin{Bmatrix} A_1 \sin\omega t \\ A_2 \sin\omega t \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

or,

$$\begin{bmatrix} -I_y\omega^2 & I_{yz}\omega^2 \\ I_{yz}\omega^2 & -I_z\omega^2 \end{bmatrix}\begin{Bmatrix} A_1 \\ A_2 \end{Bmatrix} + \begin{bmatrix} k_\theta & 0 \\ 0 & k_\psi \end{bmatrix}\begin{Bmatrix} A_1 \\ A_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

matrix addition then gives

$$\begin{bmatrix} k_\theta - I_y\omega^2 & I_{yz}\omega^2 \\ I_{yz}\omega^2 & k_\psi - I_z\omega^2 \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad \text{(A-7)}$$

The characteristic equations results from setting the determinant of the coefficient matrix to zero

$$(k_\theta - I_y\omega^2)(k_\psi - I_z\omega^2) - I_{yz}^2\,\omega^4 = 0$$

$$(I_yI_z - I_{yz}^2)\omega^4 - (k_\theta I_z + k_\psi I_y)\omega^2 + k_\theta k_\psi = 0$$

let $$I^2 = I_yI_z - I_{yz}^2 \qquad \text{(A-8)}$$

then, the natural frequencies of the system are

$$\omega^2_{\psi,\theta} = \left\{\frac{k_\theta I_z + k_\psi I_y}{2I^2}\right\} \pm \sqrt{\left\{\frac{k_\theta I_z + k_\psi I_y}{2I^2}\right\}^2 - \frac{k_\theta k_\psi}{I^2}} \qquad \text{(A-9)}$$

The amplitude ratio for the frequency ω_ψ is obtained by substituting ω_ψ into Eq. (A-7)

$$\begin{bmatrix} k_\theta - I_y\omega_\psi^2 & I_{yz}\omega_\psi^2 \\ I_{yz}\omega_\psi^2 & k_\psi - I_z\omega_\psi^2 \end{bmatrix} \begin{Bmatrix} A_{1\psi} \\ A_{2\psi} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$(k_\theta - I_y\omega_\psi^2)A_{1\psi} + I_{yz}\omega_\psi^2 A_{2\psi} = 0$$

and,

$$I_{yz}\omega_\psi^2 A_{1\psi} + (k_\psi - I_z\omega_\psi^2)A_{2\psi} = 0$$

then,

$$\frac{A_{1\psi}}{A_{2\psi}} = -\frac{I_{yz}\omega_\psi^2}{(k_\theta - I_y\omega_\psi^2)} = -\frac{(k_\psi - I_z\omega_\psi^2)}{I_{yz}\omega_\psi^2} \qquad \text{(A-10)}$$

The second subscript on A is used to indicate that the amplitudes are associated with the frequency ω_ψ. Thus, one possible solution for the equations of motion is given by the trial solution, Eq. (A-6), in which the frequency is ω_ψ and the amplitudes are related by Eq. (A-10). Another solution is possible if sin ωt is replaced by cos ωt, i. e.,

$$\begin{Bmatrix} \theta \\ \psi \end{Bmatrix} = \begin{Bmatrix} B_1 \\ B_2 \end{Bmatrix} \cos \omega t \tag{A-11}$$

A procedure similar to that for Eq. (A-6) gives frequencies identical to that of Eq. (A-9), and an amplitude ratio for ω_ψ as

$$\frac{B_{1\psi}}{B_{2\psi}} = \frac{A_{1\psi}}{A_{2\psi}} = -\frac{I_{yz}\omega_\psi^2}{k_\theta - I_y\omega_\psi^2} = -\frac{(k_\psi - I_z\omega_\psi^2)}{I_{yz}\omega_\psi^2} \tag{A-12}$$

The complete solution for a free vibration at the first natural frequency, ω_ψ, is given by

$$\theta = A_{1\psi} \sin\omega_\psi t + B_{1\psi} \cos \omega_\psi t \tag{A-13}$$

$$\psi = A_{2\psi} \sin\omega_\psi t + B_{2\psi} \cos \omega_\psi t \tag{A-14}$$

A similar procedure for the second natural frequency, ω_θ, yields

$$\theta = A_{1\theta} \sin\omega_\theta t + B_{1\theta} \cos \omega_\theta t \tag{A-15}$$

$$\psi = A_{2\theta} \sin\omega_\theta t + B_{2\theta} \cos \omega_\theta t \tag{A-16}$$

in which

$$\frac{A_{1\theta}}{A_{2\theta}} = \frac{B_{1\theta}}{B_{2\theta}} = -\frac{I_{yz}\omega_\theta^2}{k_\theta - I_y\omega_\theta^2} = -\frac{k_\psi - I_z\omega_\theta^2}{I_{yz}\omega_\theta^2} \tag{A-17}$$

The most general solution for the free vibrations of the system is given by the superposition of the motions in the two principal modes of vibration, given by Eqs. (A-13) to (A-16).

$$\theta = A_{1_\psi} \sin\omega_\psi t + B_{1_\psi} \cos\omega_\psi t + A_{1_\theta} \sin\omega_\theta t + B_{1_\theta} \cos\omega_\theta t \qquad \text{(A-18)}$$

$$\psi = A_{2_\psi} \sin\omega_\psi t + B_{2_\psi} \cos\omega_\psi t + A_{2_\theta} \sin\omega_\theta t + B_{2_\theta} \cos\omega_\theta t \qquad \text{(A-19)}$$

making use of Eqs. (A-12) and (A-17)

$$\theta = \frac{A_{1_\psi}}{A_{2_\psi}} \left\{ A_{2_\psi} \sin\omega_\psi t + B_{2_\psi} \cos\omega_\psi t \right\} + \frac{A_{1_\theta}}{A_{2_\theta}} \left\{ A_{2_\theta} \sin\omega_\theta t + B_{2_\theta} \cos\omega_\theta t \right\} \qquad \text{(A-20)}$$

$$\psi = \left\{ A_{2_\psi} \sin\omega_\psi t + B_{2_\psi} \cos\omega_\psi t \right\} + \left\{ A_{2_\theta} \sin\omega_\theta t + B_{2_\theta} \cos\omega_\theta t \right\} \qquad \text{(A-21)}$$

Eqs. (A-20) and (A-21) can be written as

$$\theta = \phi_{11} q_1 + \phi_{12} q_2 \qquad \text{(A-22)}$$

$$\psi = \varphi_{21} q_1 + \phi_{22} q_2 \qquad \text{(A-23)}$$

where,

$$\phi_{11} = \frac{I_{yz} \omega_\psi^2}{I_y \omega_\psi^2 - k_\theta} = \frac{I_z \omega_\psi^2 - k_\psi}{I_{yz} \omega_\psi^2} \qquad \text{(A-24)}$$

$$\phi_{12} = \frac{I_{yz} \omega_\theta^2}{I_y \omega_\theta^2 - k_\theta} = \frac{I_z \omega_\theta^2 - k_\psi}{I_{yz} \omega_\theta^2} \qquad \text{(A-25)}$$

$$\phi_{21} = \varphi_{22} = 1 \qquad \text{(A-26)}$$

$$q_1 = A_{2_\psi} \sin\omega_\psi t + B_{2_\psi} \cos\omega_\psi t \qquad \text{(A-27)}$$

$$q_2 = A_{2_\theta} \sin\omega_\theta t + B_{2_\theta} \cos\omega_\theta t \qquad \text{(A-28)}$$

The constants A_{2_ψ}, B_{2_ψ}, A_{2_θ}, and B_{2_θ} can be determined from the initial conditions

$$\theta(0) = \theta_o \qquad \dot{\theta}(0) = \dot{\theta}_o \tag{A-29}$$

$$\psi(0) = \psi_o \qquad \dot{\psi}(0) = \dot{\psi}_o \tag{A-30}$$

The quantities q_1 and q_2 represent the amplitudes of the motion in the two normal modes and are called the normal coordinates. A work-and -energy approach to the equations of motion shows that the multiplication of the equations of motion in the normal coordinate system by the constant $\begin{bmatrix} \phi_{11} & 1 \\ \phi_{12} & 1 \end{bmatrix}$ will uncouple the equations of motion.

Now consider the complete equations of motion, Eqs. (A-1) and (A-2), which can be written as

$$\begin{bmatrix} I_y & -I_{yz} \\ -I_{yz} & I_z \end{bmatrix} \begin{Bmatrix} \ddot{\theta} \\ \ddot{\psi} \end{Bmatrix} + \begin{bmatrix} C_\theta & 0 \\ 0 & C_\psi \end{bmatrix} \begin{Bmatrix} \dot{\theta} \\ \dot{\psi} \end{Bmatrix} + \begin{bmatrix} k_\theta & 0 \\ 0 & k_\psi \end{bmatrix} \begin{Bmatrix} \theta \\ \psi \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{A-31}$$

From Eqs. (A-22) and (A-23)

$$\begin{Bmatrix} \theta \\ \psi \end{Bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ 1 & 1 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} \tag{A-32}$$

Substituting Eq. (A-32) into Eq. (A-31) and multiplying by $\begin{bmatrix} \phi_{11} & 1 \\ \phi_{12} & 1 \end{bmatrix}$ will give uncoupled equations in the normal coordinate system

$$\begin{bmatrix} \phi_{11} & 1 \\ \phi_{12} & 1 \end{bmatrix} \begin{bmatrix} I_y & -I_{yz} \\ -I_{yz} & I_z \end{bmatrix} \begin{bmatrix} \phi_{11} & \phi_{12} \\ 1 & 1 \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} + \begin{bmatrix} \phi_{11} & 1 \\ \phi_{12} & 1 \end{bmatrix} \begin{bmatrix} C_\theta & 0 \\ 0 & C_\psi \end{bmatrix} \begin{bmatrix} \phi_{11} & \phi_{12} \\ 1 & 1 \end{bmatrix} \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{Bmatrix}$$

$$+ \begin{bmatrix} \phi_{11} & 1 \\ \phi_{12} & 1 \end{bmatrix} \begin{bmatrix} k_\theta & 0 \\ 0 & k_\psi \end{bmatrix} \begin{bmatrix} \phi_{11} & \phi_{12} \\ 1 & 1 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

matrix multiplication gives

$$\begin{bmatrix} \frac{\phi_{11}^2 k_\theta + k_\psi}{\omega_\psi^2} & 0 \\ 0 & \frac{\phi_{12}^2 k_\theta + k_\psi}{\omega_\theta^2} \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} + \begin{bmatrix} \phi_{11}^2 C_\theta + C_\psi & 0 \\ 0 & \phi_{12}^2 C_\theta + C_\psi \end{bmatrix} \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{Bmatrix}$$

$$+ \begin{bmatrix} \phi_{11}^2 k_\theta + k_\psi & 0 \\ 0 & \phi_{12}^2 k_\theta + k_\psi \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{A-33}$$

where it was assumed that $(\phi_{11}\phi_{12}C_\theta + C_\psi)$ is much smaller than either $(\phi_{11}^2 C_\theta + C_\psi)$ or $(\phi_{12}^2 C_\theta + C_\psi)$, and ω_θ and ω_ψ are given by Eq. (A-9)

Equation (A-33) is uncoupled and can be rewritten as

$$\ddot{q}_1 + \left[\frac{\phi_{11}^2 C_\theta + C_\psi}{\phi_{11}^2 k_\theta + k_\psi}\right] \omega_\psi^2 \dot{q}_1 + \omega_\psi^2 q_1 = 0 \tag{A-34}$$

$$\ddot{q}_2 + \left[\frac{\phi_{12}^2 C_\theta + C_\psi}{\phi_{12}^2 k_\theta + k_\psi}\right] \omega_\theta^2 \dot{q}_2 + \omega_\theta^2 q_2 = 0 \tag{A-35}$$

Equations (A-34) and (A-35) can be solved in the usual manner to give

$$q_1 = e^{-\delta_\psi \omega_\psi t} \left[A_\psi \cos\omega_\psi t + B_\psi \sin\omega_\psi t\right] \tag{A-36}$$

$$q_2 = e^{-\delta_\theta \omega_\theta t} \left[A_\theta \cos\omega_\theta t + B_\theta \sin\omega_\theta t\right] \tag{A-37}$$

where

$$\delta_\psi = \left[\frac{\phi_{11}^2 C_\theta + C_\psi}{\phi_{11}^2 k_\theta + k_\psi}\right] \frac{\omega_\psi}{2} \tag{A-38}$$

and,

$$\delta_\theta = \left[\frac{\phi_{12}^2 C_\theta + C_\psi}{\phi_{12}^2 k_\theta + k_\psi}\right] \frac{\omega_\theta}{2} \tag{A-39}$$

The results, Eqs. (A-36) and (A-37) can now be transformed into the θ and ψ coordinates using Equation (A-32).

$$\theta(t) = \phi_{11} \left\{ A_\psi \cos\omega_\psi t + B_\psi \sin\omega_\psi t \right\} e^{-\lambda_\psi t} + \varphi_{12} \left\{ A_\theta \cos\omega_\theta t + B_\theta \sin\omega_\theta t \right\} e^{-\lambda_\theta t} \tag{A-40}$$

$$\psi(t) = \left\{ A_\psi \cos\omega_\psi t + B_\psi \sin\omega_\psi t \right\} e^{-\lambda_\psi t} + \left\{ A_\theta \cos\omega_\theta t + B_\theta \sin\omega_\theta t \right\} e^{-\lambda_\theta t} \tag{A-41}$$

where $\lambda_\psi = \delta_\psi \omega_\psi$ and $\lambda_\theta = \delta_\theta \omega_\theta$ (A-42)

The constants in Eqs. (A-40) and (A-41) can be determined from the initial conditions, Eqs. (A-29) and (A-30).

$$A_\psi = \frac{\theta_o - \phi_{12}\psi_o}{\tilde{\phi}} \qquad B_\psi = \frac{\dot{\theta}_o - \phi_{12}\dot{\psi}_o}{\tilde{\phi}\,\omega_\psi} + \delta_\psi A_\psi$$

$$A_\theta = \frac{\phi_{11}\psi_o - \theta_o}{\tilde{\phi}} \qquad B_\theta = \frac{\varphi_{11}\dot{\psi}_o - \dot{\theta}_o}{\tilde{\phi}\,\omega_\theta} + \delta_\theta A_\theta$$

where $\tilde{\phi} = \phi_{11} - \phi_{12}$ (A-43)

The solution for the damped motion, coupled by I_{yz} is

$$\theta(t) = \varphi_{12} e^{-\lambda_\theta t} \left\{ \left(\frac{\phi_{11}\psi_o - \theta_o}{\tilde{\phi}} \right) \cos\omega_\theta t + \left[\frac{\phi_{11}\dot{\psi}_o - \dot{\theta}_o}{\tilde{\phi}\omega_\theta} + \frac{\lambda_\theta}{\omega_\theta} \left(\frac{\varphi_{11}\psi_o - \theta_o}{\tilde{\varphi}} \right) \right] \sin\omega_\theta t \right\}$$
$$+ \varphi_{11} e^{-\lambda_\psi t} \left\{ \left(\frac{\theta_o - \varphi_{12}\psi_o}{\tilde{\phi}} \right) \cos\omega_\psi t + \left[\frac{\dot{\theta}_o - \varphi_{12}\dot{\psi}_o}{\tilde{\varphi}\omega_\psi} + \frac{\lambda_\psi}{\omega_\psi} \left(\frac{\theta_o - \phi_{12}\psi_o}{\tilde{\phi}} \right) \right] \sin\omega_\psi t \right\} \tag{A-44}$$

$$\psi(t) = e^{-\lambda_\psi t} \left\{ \left(\frac{\theta_o - \phi_{12}\psi_o}{\tilde{\phi}} \right) \cos\omega_\psi t + \left[\frac{\dot{\theta}_o - \phi_{12}\dot{\psi}_o}{\tilde{\varphi}\omega_\psi} + \frac{\lambda_\psi}{\omega_\psi} \left(\frac{\theta_o - \phi_{12}\psi_o}{\tilde{\varphi}} \right) \right] \sin\omega_\psi t \right\}$$
$$+ e^{-\lambda_\theta t} \left\{ \left(\frac{\varphi_{11}\psi_o - \theta_o}{\tilde{\phi}} \right) \cos\omega_\theta t + \left[\frac{\varphi_{11}\dot{\psi}_o - \dot{\theta}_o}{\tilde{\phi}\omega_\theta} + \frac{\lambda_\theta}{\omega_\theta} \left(\frac{\varphi_{11}\psi_o - \theta_o}{\tilde{\phi}} \right) \right] \sin\omega_\theta t \right\} \tag{A-45}$$

where φ_{11}, ϕ_{12} and $\tilde{\phi}$ are given by Eqs. (A-24), (A-25) and (A-43) respectively.

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13. ABSTRACT
Wind-off, bench tests on a wind tunnel model that had freedom to pitch, and yaw, but no freedom to roll indicated that the pitching and yawing motions were inertially coupled. An order of magnitude analysis showed that this coupling was due to the product of inertia term I_{yz}. The inertially coupled equations of motion are solved, and a method is presented for determining the magnitude and sign of I_{yz} from recordings of the model motion. The method used to dynamically balance the model is described, and experimental results are presented that verify this method.

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